

Chapter 3 The particle-like nature of electromagnetic waves

Maxwell's equations

Clerk Maxwell (1831-1879)

Microscopic form

① $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ (Gauss's law) $\vec{E}$ = electric field vector
 ρ = total charge density
 ϵ_0 = permittivity of free space (electric constant)
 or $\oint_V \vec{E} \cdot d\vec{A} = \frac{Q(V)}{\epsilon_0}$
meaning ^{static} How charges emit an electric field

② $\vec{\nabla} \cdot \vec{B} = 0$ (Gauss's law for magnetism) $\vec{B}$ = magnetic field vector
 or $\oint_V \vec{B} \cdot d\vec{A} = 0$
meaning ^{static} How currents produce magnetic fields

③ $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday's law of Induction)
meaning how changing magnetic field induces an electric field

④ $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ (Ampere's law) $\vec{J}$ = current density
 μ_0 = permeability of free space (magnetic constant)
meaning How currents and time-varying electric fields induce magnetic fields

If we eliminate charges and currents, one solution to the Maxwell equations is

$$\vec{E} = E_0 \hat{x} \sin(kz - \omega t)$$

$$\vec{B} = B_0 \hat{y} \sin(kz - \omega t)$$

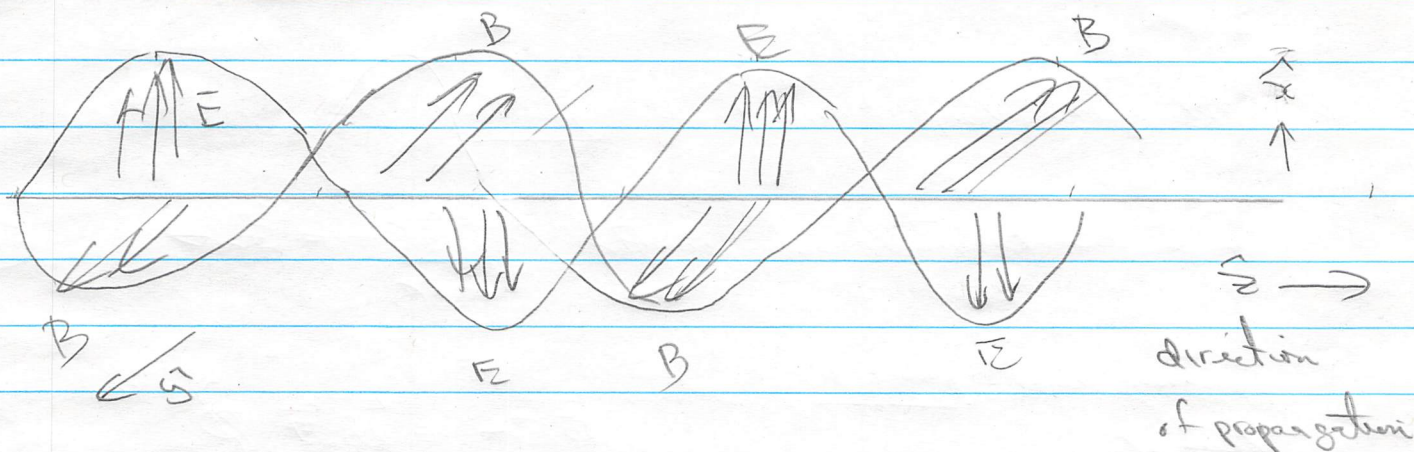
$$\frac{\omega}{k} = c; \quad B_0 = \frac{E_0}{c}$$

That describes a disturbance propagation along the z axis, going with velocity $c = (\epsilon_0 \mu_0)^{-1/2}$ (k = wave number, ω = frequency)

that is generated and perpetuated by oscillating electric and magnetic fields

time varying $\vec{E} \rightarrow \vec{B}$
time varying $\vec{B} \rightarrow \vec{E}$

Picture



Energy flux (Poynting vector, named after John Henry Poynting (1852-1914))

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

Successes

Maxwell's equations are incredibly successful at describing "light" in many situations

Circuits — electronics, light / heat generation

dielectrics — polarization, reflection, absorption

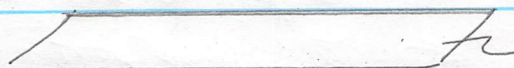
magnets — motors

Failings

Photoelectric effect

} measurements by
Heinrich Hertz (1857-1894)
in 1887

light



metal plate

in a metal plate,

"free" electrons are

bound by a "work function"

amounting to a few eV

e.g. for Cu, 4.7 eV

Stainless steel	4.4 eV
Gold	5.1 eV
Silver	4.26 eV

The theoreticians of the time theorized that

(energy)

- 1) Any wavelength would liberate electrons if the electrons were given enough time to absorb (i.e. time delay $\rightarrow$ all light would liberate e^-)

- 2) If increase energy of the light, $K_e \uparrow$ as well
(intensity)

Contradictory observations

- 1) If $\lambda > \lambda_0$ (wavelength of light) no e^- , no matter how intense
- 2) If $\lambda < \lambda_0$, e^- appear, instantly, no delay
- 3) If the intensity of light increases, the # of e^- increases, not their K_e

Resolution

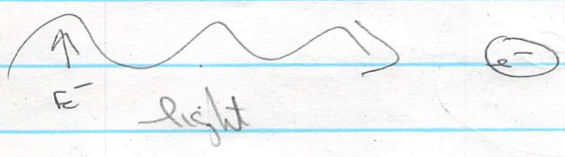
Albert (1879-1955) NP 1921

Einstein: "Em waves are really made up of single particles called photons"

We know now that the particle-like properties of waves only exhibit their particle like nature when their wavelength is of the order of, or smaller, than the objects they are interacting with

The Compton Effect

Classical picture of light shining on a free electron



oscillating electric field causes the e^- to wiggle up and down

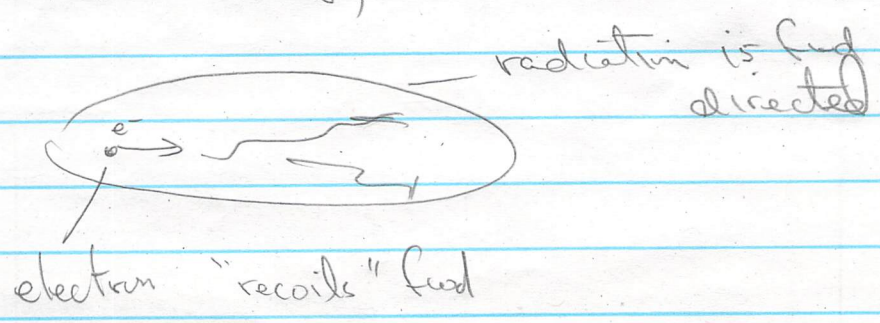


This causes the e^- to re-radiate (Maxwell's equations)



NP 1906
called Thomson scattering
"53" Joseph John Thomson (1856-1940)
the e^- is not pushed into the fwd direction

However, at higher energy



The final trajectory of the electron proves that the photon carries momentum and transmits some of it to the electron according to the laws of $\text{CoE} \& \text{CoM}$

Blackbody radiation

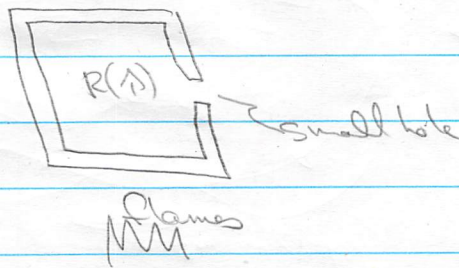
If you heat up an object (and it doesn't catch fire), what does it do?

(Turn on an electric burner on the stove and turn off the lights)

It glows!

This is called "blackbody" radiation.

The "radiance" is the energy density in an interval $\lambda \rightarrow \lambda + d\lambda$ inside a "blackbody" radiator



Ludwig Boltzmann (1844-1906)

from classical $e \& m$

Boltzmann's constant

$$R(\lambda) = \frac{2\pi^5 k_B^4}{15} T^4$$

λ^4

$\leftarrow$ temperature in Kelvin

degrees Kelvin
 $\downarrow$

Note means $8.617343(15) \times 10^{-5}$
 $8.617343 \times 10^{-5} \pm 0.000015 \times 10^{-5}$

$$k_B = 8.617343(15) \times 10^{-5} \text{ (eV/K)}$$

This calculation, which is based on Maxwell's equations, has an interesting prediction

The total Radiant energy emitted by "anything" is

$$R_T = \int_0^{\infty} d\lambda R(\lambda) \rightarrow \infty !$$

This was an annoying chick in the armor of Classical e.p.m.

Max Planck solved the problem by saying that the photon energy is quantized (a la Einstein)

Planck's constant

So that $E_\gamma = \frac{hc}{\lambda}$ speed of light

$\lambda \leftarrow$ wavelength

We will be able to derive this (later in the course!) that $R(\lambda)$ really is

$$R(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \quad (1)$$

Planck's Constant $h = 4.13566733(10) \times 10^{-15} \text{ eV s} \leftarrow \text{seconds}$

$\uparrow$
Max Planck (1858-1947) $\hbar = \frac{h}{2\pi} = 6.5211899(16) \times 10^{-16} \text{ eV s}$
NP 1988

① reveals that, as $\lambda \rightarrow 0$ $R(\lambda) \rightarrow 0$

and R is finite, not only resolving the shortcoming, but also providing an experimental determination of h .

Classical limit

The classical limit of ① is obtained by

$$\lim_{\lambda \rightarrow \infty} R(\lambda) = \frac{2\pi c k T}{\lambda^4}$$

in agreement with the classical result

$$\text{Also } \lim_{\lambda \rightarrow \infty} R(\lambda) = \frac{2\pi c k T}{\lambda^4}$$

so that classical and "quantum" physics agree for large λ (expected)

